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ОДИНАКОВЫЙ ПО МЕСТООБИТАНИЯМ ПРИНЦИП РОСТА НА ПРИМЕРЕ РОСТА В ВЫСОТУ У ЕЛИ И ДРУГИХ ДРЕВЕСНЫХ ВИДОВ

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A site-independent growth principle for tree heights is presented. Under «normal conditions» this principle leads to identical growth data sequences for different sites if one uses the dimensionless units relative to the maximum of increases. These units are the time (age), t_m, at which the maximal increase appears and the tree height, X_m, which is reached at that time. Transition to these units corresponds formally to the transformation t' = t/t_m and X' = X/X_m. The identical behaviour of the growth curves is demonstrated for spruce and red oak. For the sake of comparison, preliminary results on the growth behaviour of other tree species are presented also.

Key words: *growth curves, relative units, invariance principle.*

TRANSFORMATION TO A SITE-INDEPENDENT GROWTH CURVE

This contribution is based on former results [1, 6]. A universal growth principle for the height growth of trees and stands states that (for equal forest management and constant site conditions) all growth data sequences get identical independent of their sites (and roughly independent of the tree species also) if the dimensionless units are used for age *t* and height *X* which are related to the parameters t_m and X_m characterizing the maximum of increases. The new variables are

$$t' = \frac{t}{t_m},\ X' = \frac{X}{X_m}, \qquad (1)$$

where t_m, is the age when the maximal increase appears and X_m, is the height that is reached at this moment. Therefore, t_m and X_m represent the transformation parameters.

This transformation has been already used elsewhere for another purpose [7]. To apply transformation (1) in a real case one needs to know the particular values of t_m and X_m for that case. The data for *X(t)* are mostly given only for t-values with regular gaps of 10 years. Therefore, one has to interpolate these data within the 10-year intervals in order to determine the transformation parameters t_m and X_m. To do so, the height increase data have been fitted for the total period of $0 < t < 2t^e_m$ (with t^e_m being a rough estimate of t_m) by a polynomial:

$$\frac{dX}{dt} = a\cdot t^3 + b\cdot t^2 + c\cdot t + d\ . \qquad (2)$$

Under assumption that the position of the maximum of polynomial (2) is a suitable approximation for the position of the maximum for the real increases, one can calculate this position as an extremal point of the polynomial:

$$t_m = -\frac{b}{3a} \pm \sqrt{\left(\frac{b}{3a}\right)^2 - \frac{c}{3a}}\ . \qquad (3)$$

The second transformation parameter X_m can thereafter be obtained in a similar way. The height data have also been fitted by a polynomial, and its value at the argument $t = t_m$ yields the value of X_m:

$$X_m = X\left(t_m\right) = a'\cdot t_m^3 + b'\cdot t_m^2 + c'\cdot t_m + d'\ . \qquad (4)$$

RESULTS FOR TRANSFORMATIONS OF VARIOUS GROWTH DATA SEQUENCES

a) Heights of spruce

Data documented in «Yield Tables» for the growth of spruce are considered first. Fig.1 shows data sequences of the mean heights and also for the mean values of the heights of 10 single trees for 3 different site classes [4].

In Fig. 2, the 3 transformed growth data sequences of the mean heights for the 3 site classes already shown in Fig. 1b are compared with 9 transformed growth data sequences of different «stand qualities» from other areas [2]. When transformed, the different growth data sequences mainly agree. The small differences could be caused, e.g., by changes in «site qualities» after 1915 or by imprecise determination of the transformation parameters t_m and X_m.

b) Heights of other tree species

Fig. 3 shows transformed growth data of red oak in 3 site classes. Again, these growth data sequences behave almost identically. For the purpose of comparison the transformed data sequence of spruce (for the best «stand quality» M40 [2]) is shown here too. A remarkable agreement between the transformed growth data sequences of read oak and that of spruce can be seen.

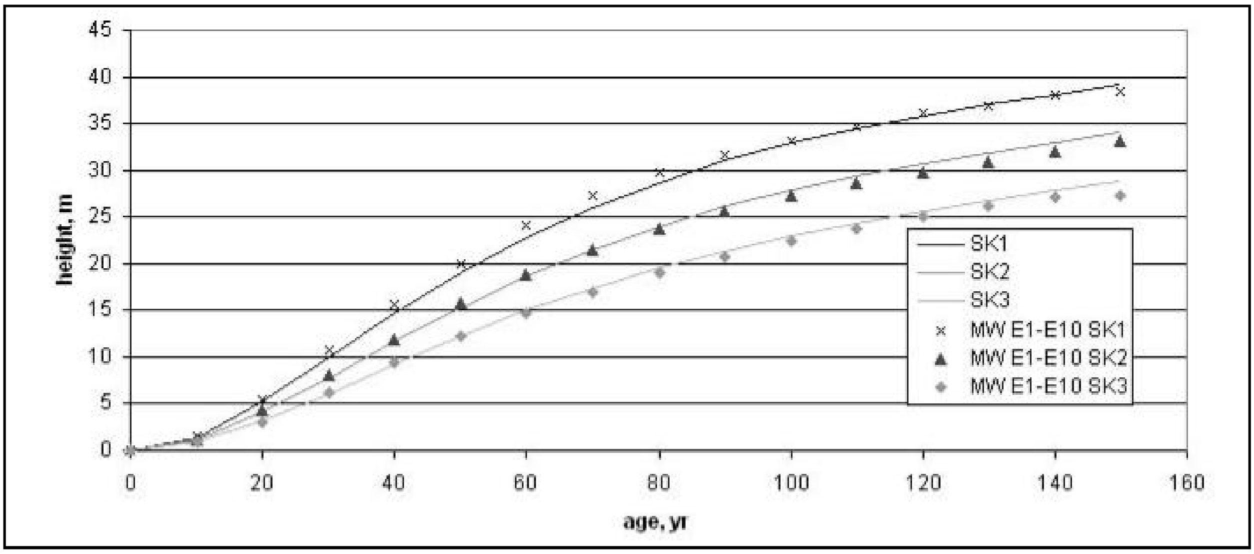

Fig. 1a. Growth data series for mean heigths and for mean values of 10 single trees in 3 different site classes

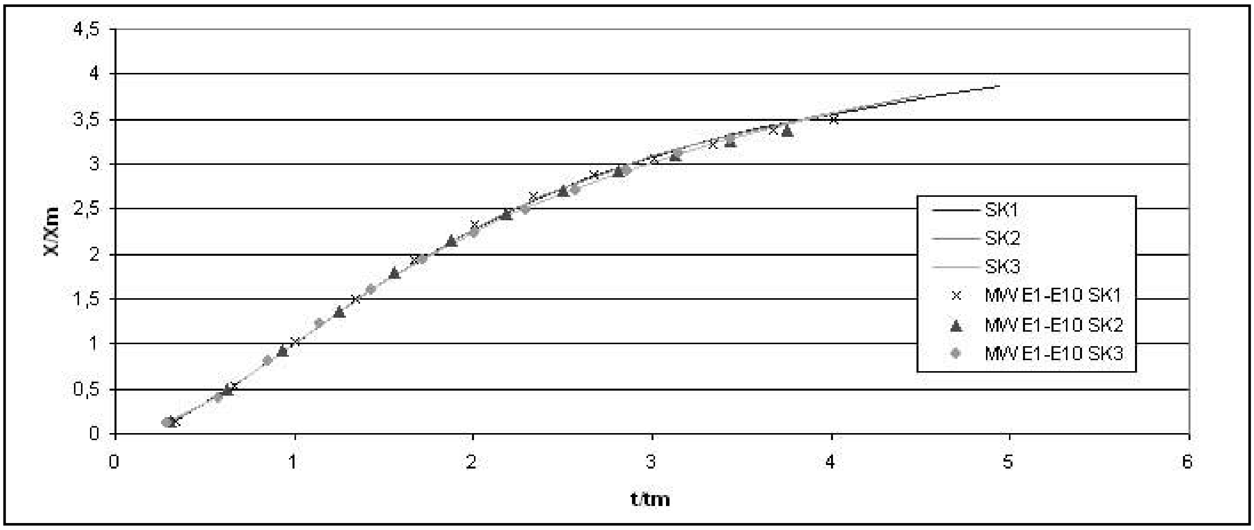

Fig. 1b. Transformed growth data sequences for mean heigths and for mean values of 10 single trees for 3 different site classes

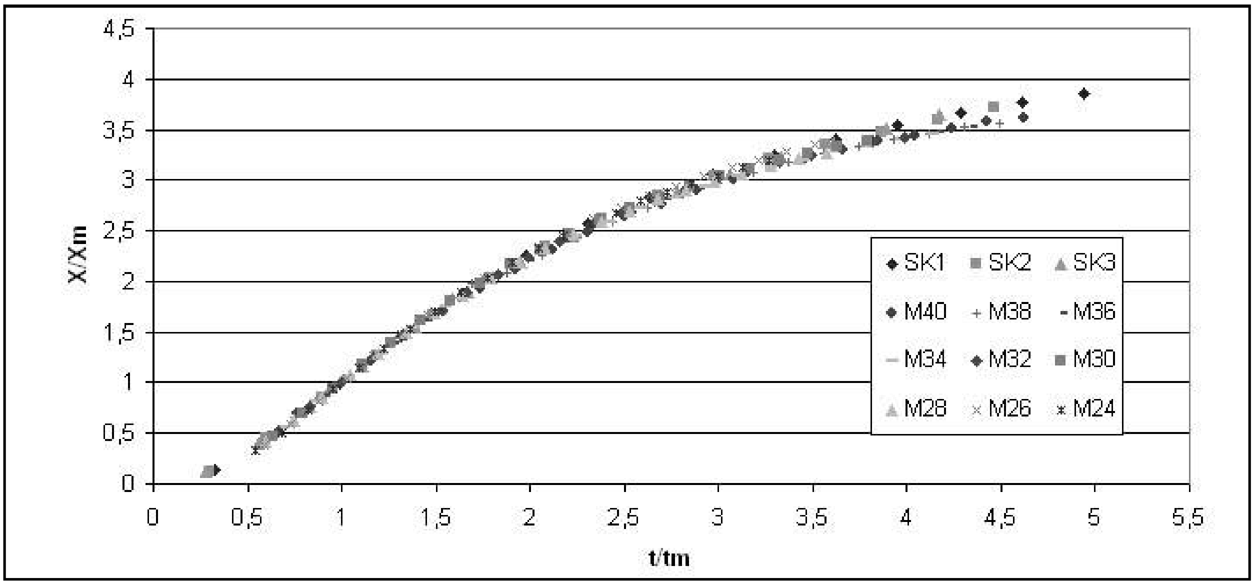

Fig. 2. Transformed growth data sequences for different site classes and stend qualities

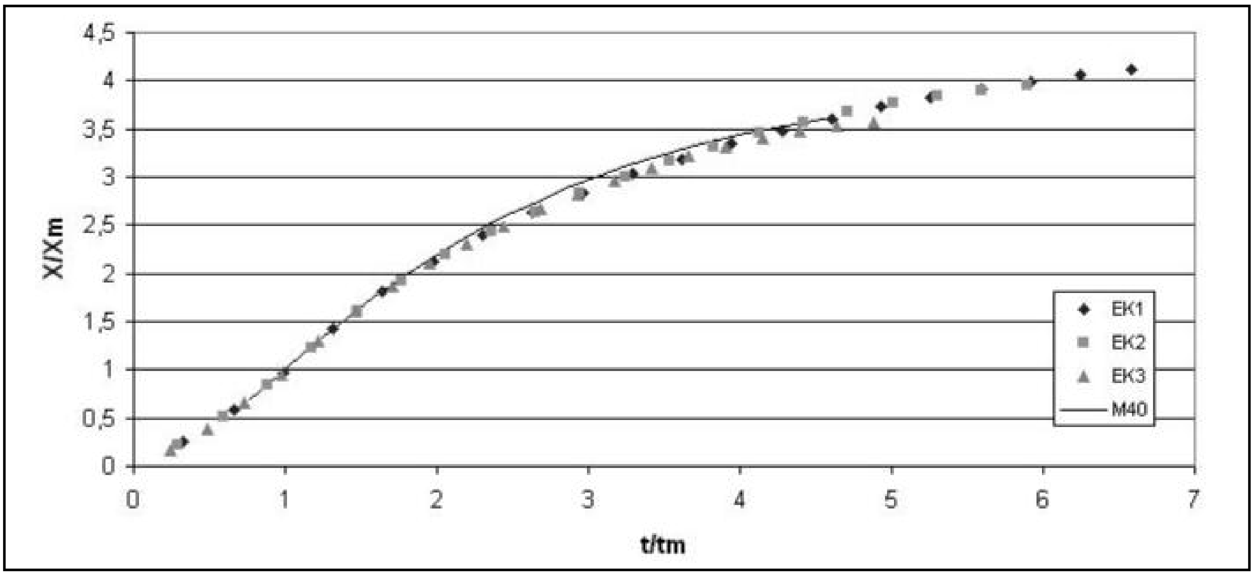

Fig. 3. Transformed growth data sequences of red oak for different site classes and of spruce for the best site class

This result has stimulated the study of transformed growth data for further tree species. Preliminary results for such transformed growth data are shown in Fig. 4 for the best «site qualities» of spruce, red oak, fir, copper beech, and Douglas fir. The agreement is not so good as in the case of spruce and red oak, but similarities are nevertheless visible. For two additional tree species (larch and oak) the similarity with the general trend is worse (deviations up to 10%). There may be a number of reasons for the discrepancy, e.g., inaccurate determination of the transformation parameters t_m and X_m or changes in the «site quality», but also differences in the growth mechanisms of different tree species. The results described have been obtained on the basis of data analysis alone, without any growth models. Models may be used for further interpretation. For example, the spruce growth can be interpreted by means of a growth process model originated from Werner Mende [5]. Application of that model to the data described here reveals the following relation to be approximately true for all considered tree species:

$$X_L = 5 \cdot X_m, \qquad (5)$$

with: X_L = the theoretical maximal limit of the height X for the given site and

X_m = the height at the age when the maximum of increases appears.

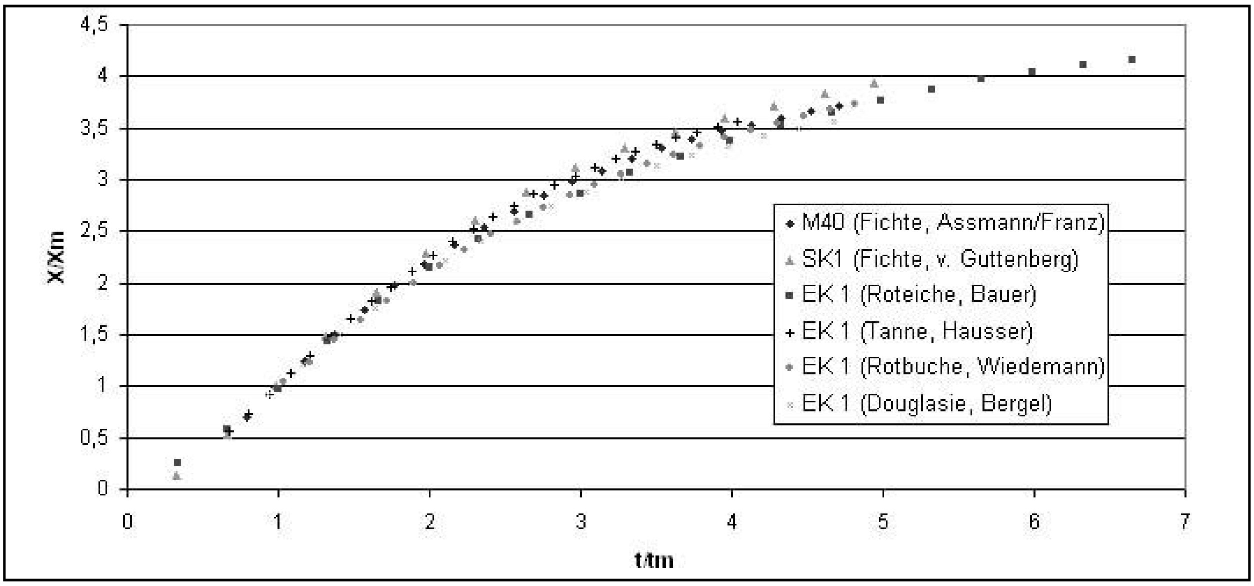

Fig. 4. Transformed growth data sequences for various tree species

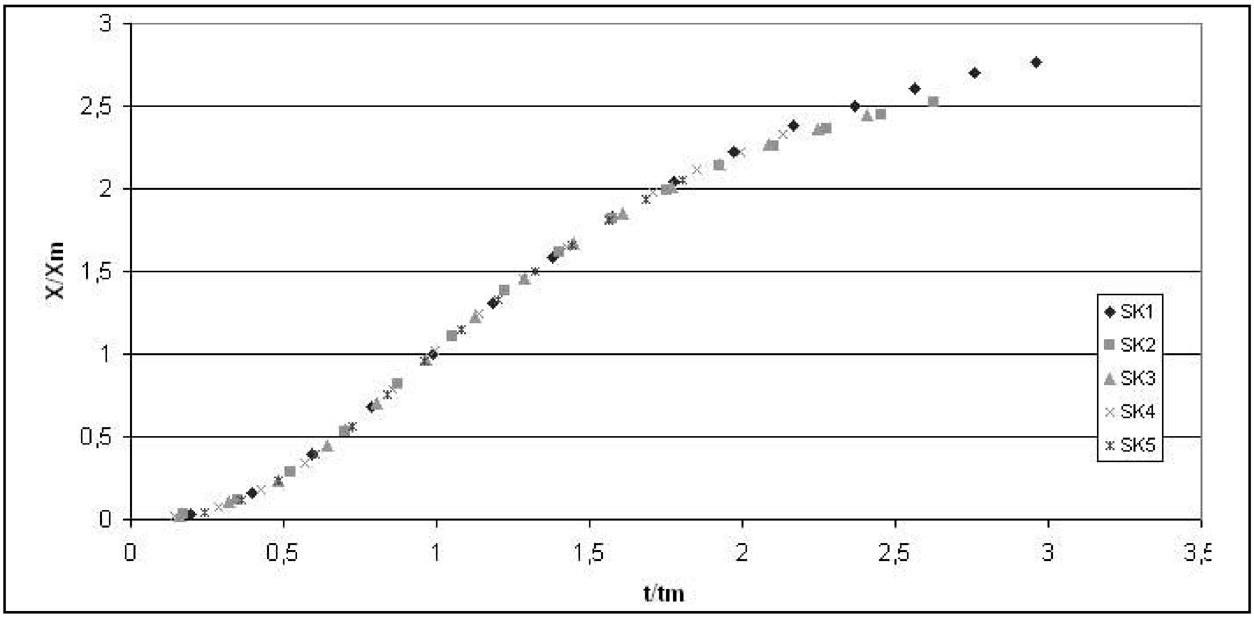

Fig. 5. Transformed growth data sequences for wood mass per ha, SK1-SK5

c) Another growth index

As a first attempt to study the behaviour of other growth indexes, the growth of wood mass per ha has been considered. Fig. 5 shows the transformed data sequences of this growth index in 5 site classes for spruce [4]. Again, the behaviour of these sequences for different site classes agrees quite well. In this case, the maximum of increases appears considerably later than in the case of height growth, with more data points before the maximum to provide for more precise determination of the transformation parameters t_m and X_m.

CONCLUSION

The growth data sequences for a number of tree species exhibit identical behaviour irrespective of the site class when the age is measured in the relative units of t_m and the height in the relative units of X_m, the both parameters being defined with regard to the moment when the growth is maximal and to the measured value at that moment, respectively. In the formal terms, this reduces to the transformation: $t' = t/t_m$, $X' = X/_m$, whereupon growth data obtained from different sites can be checked and combined.

To describe the height growth it is sufficient to know the values of t_m and X_m. These values thus characterize the site in terms of stand growth, and their accurate identification becomes important. Consequently, more measurements are required at the ages in a vicinity of t_m .

The comparison of transformed growth data sequences among different tree species does not reveal such a good agreement as it does for different sites of the same tree species. But there is a rough agreement in this case, too. Also, the transformation method can be applied for other growth indexes, thus being useful for prognoses in forest science and management.

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UNIVERSAL, SITE-INDEPENDENT GROWTH PRINCIPLE FOR THE HEIGHT GROWTH OF SPRUCE AND OTHER TREE SPECIES

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